

# Advanced Math

9a-1

## Sequences and Summation

### Notation

Discrete function - a function whose domain is (usually)  $\mathbb{N}$  and whose outputs are a collection of separate elements.

Write the first five terms of the sequence. (pg 716)

1)  $a_n = 2n + 1$

↑  
input,  
 $n \in \mathbb{N}$

$n=1, a_1 = 2(1) + 1 = 3$   
 $a_2 = 5$   
 $a_3 = 7$   
 $a_4 = 9$   
 $a_5 = 11$

Recursively Defined Function - A function whose output is dependent on the previous output of the same function. The first output is called the seed.

Write the first five terms of the sequence defined recursively.

25)  $a_1 = 15$ ,  $a_{k+1} = a_k - 4$

$a_1 = 15$   
 $a_2 = a_1 - 4 = 15 - 4 = 11$   
 $a_3 = 7$   
 $a_4 = 3$   
 $a_5 = -1$

Write an expression for the most apparent  $n$ th term of the sequence (assume  $n$  begins at 1).

47)  $1, 4, 7, 10, 13, \dots$

$a_n = -2 + 3n$

recursive

$a_{k+1} = a_k + 3$

$a_1 = 1$

59)  $1, -1, 1, -1, 1, \dots$

$a_n = (-1)^{n+1}$   
 sign switcher

$a_1 = 1$

$a_{k+1} = -a_k$

Factorial -  $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$   
 $x! = x(x-1)(x-2) \dots (2)(1)$   
 $0! = 1$  (by definition)

Simplify the ratio factorials.

41)  $\frac{10!}{8!} = \frac{10 \cdot 9 \cdot 8!}{8!} = 90$

We can stop writing numbers and replace with a factorial at any time, because the factorial counts down the rest of the numbers.

Summation and Sigma notation -

$\sum_{n=x}^y a_n$

↑ end #  
 ↓ start #  
 rule  
 remember,  $n \in \mathbb{N}$ , the natural numbers  $\{1, 2, 3, \dots\}$

Find the sum.

65)  $\sum_{i=0}^4 (2i+1) = (2 \cdot 0 + 1) + (2 \cdot 1 + 1) + (2 \cdot 2 + 1) + (2 \cdot 3 + 1) + (2 \cdot 4 + 1)$   
 $= 1 + 3 + 5 + 7 + 9 = 25$

Use Sigma notation to write the sum.

81)  $\frac{1}{3(1)} + \frac{1}{3(2)} + \frac{1}{3(3)} + \dots + \frac{1}{3(9)}$

$\sum_{n=1}^9 \frac{1}{3n}$

start number (points to 1)  
end number (points to 9)  
since this is the only changing value, we assign it a variable (points to n)

Assignment:

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2-22 every 4th,

26, 28,

40-60 even,

66-80 every 4th,

82-90 even.